Coating Line Optimization Problem

In many cases, automated coating lines are built in such a way that racks and/or trays of various types - carrying the different pieces which shall be treated - are moving trough the sections of the facility periodically in rounds, very similar to a cableway.

Once a rack has finished a round, the coated pieces are removed and the rack is equipped with new, untreated pieces, ready to enter the subsequent round. But keeping the racks and their order unchanged is just one possible (and the ideal) situation. In other cases, as the racks are of different type, it is necessary to add or remove racks, depending on whether the next round needs new rack types, no longer requires certain rack types or a different order of the racks is requested in order to optimally support the subsequent production processes.

Problem Definition

The Coating Line Optimization Problem is defined as follows:

Given two fixed rounds of racks $R_1 = [r_1^1, r_1^2, \dots, r_1^n]$ and $R_2 = [r_2^1, r_2^2, \dots, r_2^m]$ of lengths n and m with each of the racks $r_{[1,2]}^x$ having assigned a certain rack type $t[r_{[1,2]}^x]$, decide which racks are to be removed from Round R_1 and which racks have to be inserted in Round R_2 in order to transform Round R_1 into Round R_2.

Note that the respective order of the rack types in the rounds matters, but racks of identical rack type are indistinguishable from each other. Apart from maintaining the given order of the groups of identical rack types, the goal is to keep the total number of removed and inserted racks as small as possible because each of these operations slows down the production process.

Example

Suppose we are given the following two rounds:

$$R_1 = [r_1^1, r_1^2, r_1^3, r_1^4] \quad \text{with } t[r_1^1] = A,\ t[r_1^2] = B,\ t[r_1^3] = A,\ t[r_1^4] = C$$
$$R_2 = [r_2^1, r_2^2, r_2^3, r_2^4, r_2^5] \text{ with } t[r_2^1] = B,\ t[r_2^2] = A,\ t[r_2^3] = A,\ t[r_2^4] = D,\ t[r_2^5] = E$$

The three optimal solutions with a cost of 5 moves are the following:

- $remove_{R_1} = \{r_1^1, r_1^4\}$ $\quad insert_{R_2} = \{r_2^2, r_2^4, r_2^5\}$
- $remove_{R_1} = \{r_1^1, r_1^4\}$ $\quad insert_{R_2} = \{r_2^3, r_2^4, r_2^5\}$
- $remove_{R_1} = \{r_1^2, r_1^4\}$ $\quad insert_{R_2} = \{r_2^1, r_2^4, r_2^5\}$

Explanation: Three of the moves are trivial, namely those concerning the rack types C, D and E. They do not occur in the other round and so they have to be removed (C) or inserted (D and E). The three solutions then arise because the first two positions in the rounds are swapped between Round R_1 and Round R_2, so we can either keep A while inserting and removing B or do the same with B (where we have two choices for the position where A is inserted).